Chapter 3
Section 3.1

Recap and Then Some

In Section 2.5 we discussed inverses of functions. In particular, we determined that a function is invertible if and only if it is one-to-one.

Strategy: Let f be a one-to-one (invertible) function. Suppose that $(x_1, f(x_1))$ and $(x_2, f(x_2))$ are two ordered pairs in f. If we suppose that $f(x_1) = f(x_2)$ then what must the relationship between x_1 and x_2 be?

Exercise: Show that $j(x) = \frac{x+3}{x-5}$ is invertible and find $j^{-1}(x)$.

Example: Consider the quadratic function $g(x) = x^2 - 6x + 13$ and the function $f(x) = \sqrt{x-4} + 3$.

a) Determine the domain of g and f.

b) By completing the square, write g as a member of the transformation family of x^2.

c) Show that g is not invertible (one-to-one).

d) Show that $f(x)$ is invertible and find its inverse.

e) What is the domain of f^{-1}?

Question: How is it possible that g is not invertible, but f is?

Further Question: If we restrict the domain of g to a subset of its former self, then we can make g a one-to-one function **on the new domain.** What should this new domain be?

Quadratic Functions

A quadratic function is a function defined by a quadratic equation. Another way of classifying quadratic functions is as the set of functions in the transformation family of $y = x^2$ which are not constant functions (i.e. the set of functions of the form $f(x) = a(x-h)^2 + k$ with a, h and k real numbers with $a \neq 0$ or $\{f(x) : f(x) = a(x-h)^2 + k \text{ and } a, h, k \in \mathbf{R} \text{ and } a \neq 0\}$).

Exercise: By completing the square, write $f(x) = x^2 + 6$ as a member of the transformation family of $y = x^2$ and graph $f(x)$.

Graphs of Quadratic Functions

If we consider a quadratic function of the form $f(x) = a(x-h)^2 + k$ then we can classify how the graph of $f(x)$ looks just be consider the constants a, h and k. For instance, if $a > 0$ then $f(x)$ **opens upward**; but if $a < 0$ then the graph **opens downward.** Remember also that h determines the amount of horizontal translation and k determines the amount of vertical translation. Because of this, we say that (h, k) is the **vertex** of $f(x)$ and that $f(x) = a(x-h)^2 + k$ is the **vertex form** of f. We can also determine that f has an **axis of symmetry** on the line $x = h$. **Exercise:** Find the vertex of the following quadratic functions.

a) $f(x) = -2x^2 - 4x + 3$.

b) $g(x) = 2x^2 - 4x + 9$.

Definition: The **minimum value** of a function f is the least value $y = f(x)$ for $x \in dom f$. The **maximum value** of the function f is the greatest value $y = f(x)$ for $x \in dom f$.

The domain of every quadratic function is all real numbers, $(-\infty, \infty)$. The range is determined by the second coordinate of the vertex. If f opens upward then its range is $[k, \infty)$ and k is the minimum value of f. If f opens downward then its range is $(-\infty, k]$ and k is the maximum value of f.

Question: What is the domain and range of f and g in the exercise above?

Quadratic Inequalities

When solving quadratic inequalities there are two different methods you can use. You can either graph it (which we have seen before) or you can use the **test-point method**. **Exercise:** Use the graphical method for solving quadratic inequalities to solve the following inequalities.

a) $x^2 - x > 6$.

b) $(x+3)^2 + 2 < 6$.

Exercise: Use the test-point method to solve the following quadratic inequalities.

a) $2x^2 - 4x - 9 < 0$.

b) $w^2 - 4w - 12 \geq 0$.

Applications of Maximum and Minimum

Exercise: A ball is tossed straight upward with an initial velocity of 80 feet per second from a rooftop that is 12 feet above ground level. The height of the ball in feet at time t in seconds is given by $h(t) = -16t^2 + 80t + 12$. Find the maximum height above ground level for the ball.

Exercise: If 100 m of fencing will be used to fence a rectangular region, then what dimensions for the rectangle will maximize the area of the region?

Exercise: If 100 m of fencing will be used to fence three sides of a rectangular region (because one side of the region is enclosed by the side of a house) then what dimensions for the rectangle will maximize the area of the region?